

Mathematics 1A, Fall 2010 — M. Christ
Practice Midterm Exam #2

Show work and/or reasoning where indicated. Draw boxes around your final answers to receive credit.

(1) Calculate $f'(x)$, using any method from this course. Show your steps.

(1a) $f(x) = x^{\ln(x)}$

(1b) $f(x) = \sqrt{\arcsin(x)}$

(1c) $f(x) = x^{-2}e^{3x}$

(1d) If $x^4 - 3xy + 2y^3 = 12$, find dy/dx when $(x, y) = (2, 1)$.

(2) Evaluate the following limits. Show each of your steps clearly, but you need not justify those steps in words.

(2a) $\lim_{x \rightarrow \infty} \frac{x^{1/3}}{\ln(x)}$

(2b) $\lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x^2}$

(3a) Find the maximum value of $f(x) = x(x-3)^2$ on $[-1, 5]$, and determine all points in this interval where that value is attained. You may use: $f'(x) = 3(x-1)(x-3)$. Show all steps; you will be graded on these steps, not merely on your answer.

(3b) If x and y are positive numbers and $xy^2 = 3$, what is the minimum possible value of $9x + y$? Show your steps.

(4) Short answer questions.

(4a) Define: The graph of f has a slant asymptote as $x \rightarrow -\infty$.

(4b) Suppose that f and f' are differentiable functions on an interval (a, b) , $c \in (a, b)$, and $f'(c) = 0$. What can one conclude if $f''(c) > 0$? What if $f''(c) = 0$? (Answer both questions.)

(4c) Let $f(x) = e^x$. What is the equation for the linearization (also known as the linear approximation) of f at 3?

(5) Show that if $x > 1$, then $\ln(x) < x - 1$.